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Pion–nucleon interaction and the strangeness content of the nucleon

M.E. Sainio

[1] Helsinki Institute of Physics, P.O. Box 64, 00014 University of Helsinki, Finland
[2] Department of Physical Sciences, University of Helsinki, Finland

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Abstract. A brief review of the pion-nucleon sigma-term is given.

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1 Introduction

Sigma-terms are proportional to the matrix elements

$$\langle A|m_q\bar{q}q|A\rangle \ \ ; q=u,d,s \ \ ; \ A=\pi,K,N$$

of scalar quark currents in the framework of QCD. These matrix elements are of interest, because they are related

- to the mass spectrum,
- to scattering amplitudes through Ward identities,
- to the strangeness content of A,
- to quark mass ratios.

In the following the status of the πN system is considered and the implications to the strangeness content in the nucleon are outlined.

2 The πN sigma-term

The pion-nucleon sigma-term is a measure of explicit chiral symmetry breaking in QCD and it is defined as

$$\sigma=\frac{\hat{m}}{2m}\langle p|\bar{u}u+\bar{d}d|p\rangle, \qquad \hat{m}=\frac{1}{2}(m_u+m_d), \qquad (1)$$

which is the $t=0$ value of the nucleon scalar form factor $\sigma(t)$

$$\bar{u}'\sigma(t)u=\hat{m}\,\langle p'|\bar{u}u+\bar{d}d|p\rangle, \ \ t=(p'-p)^2, \qquad (2)$$

i.e. $\sigma=\sigma(t=0)$. The nucleon mass is denoted by m.

The strangeness content of the proton can be defined as

$$y=\frac{2\,\langle p|\bar{s}s|p\rangle}{\langle p|\bar{u}u+\bar{d}d|p\rangle} \qquad (3)$$

(the OZI rule would imply y=0).

Algebraically σ can be written in the form

$$\sigma=\frac{\hat{m}}{2m}\frac{\langle p|\bar{u}u+\bar{d}d-2\bar{s}s|p\rangle}{1-y}, \qquad (4)$$

where the numerator is proportional to the octet breaking piece in the hamiltonian. To first order in SU(3) breaking we have now

$$\sigma\simeq\frac{\hat{m}}{m_s-\hat{m}}\frac{m_\Xi+m_\Sigma-2m_N}{1-y}\simeq\frac{26\text{ MeV}}{1-y}, \qquad (5)$$

where the quark mass ratio takes the value

$$\frac{m_s}{\hat{m}}=2\,\frac{M_K^2}{M_\pi^2}-1\simeq 25 \qquad (6)$$

in terms of the kaon and pion masses. Chiral perturbation theory (ChPT) allows us to determine the combination

$$\hat{\sigma}=\sigma(1-y) \qquad (7)$$

from the baryon spectrum.

For $\hat{\sigma}$ we have:

- 26 MeV (leading order)
- 35±5 MeV, $\mathcal{O}(m_q^{3/2})$ [1]
- 36±7 MeV, $\mathcal{O}(m_q^2)$ [2]
- 33±3 MeV, $\mathcal{O}(m_q^2)$ [3],

where the difference in the last two determinations is the regularization method used, dimensional regularization [2] or cut-off [3].

With the help of the Feynman-Hellmann theorem the sigma-term can be extracted from the nucleon mass

$$\sigma=\hat{m}\frac{\partial m}{\partial\hat{m}}. \qquad (8)$$

Equivalently, employing $M^2=2\hat{m}B$,

$$\sigma=M^2\frac{\partial m}{\partial M^2}, \qquad (9)$$

where B is the scalar vacuum condensate. The quark mass expansion of the nucleon mass [4]

$$m = m_0 + k_1 M^2 + k_2 M^3 + k_3 M^4 \ln \frac{M^2}{m_0^2} + k_4 M^4 + \mathcal{O}(M^5) \tag{10}$$

yields for σ

$$\sigma = k_1 M^2 + \frac{3}{2} k_2 M^3 + k_3 M^4 \{2 \ln \frac{M^2}{m_0^2} + 1\} + 2k_4 M^4 + \mathcal{O}(M^5). \tag{11}$$

Here the factors k_i contain the low-energy constants appearing in the respective chiral order. Numerically

$$\sigma = (75 - 23 - 7 + 0) \text{ MeV} = 45 \text{ MeV},$$

where the $\mathcal{O}(M^2)$ term, k_1, has been fixed by taking σ to have the value 45 MeV [5].

3 The πN amplitude

To relate the sigma-term discussion to the scattering information the standard representation for the πN amplitude is adopted

$$T_{\pi N} = \bar{u}'[A(\nu, t) + \frac{1}{2}\gamma^\mu (q + q')_\mu B(\nu, t)]u. \tag{12}$$

The definition of the crossing variable is

$$\nu = \frac{s - u}{4m} = \omega + \frac{t}{4m}, \tag{13}$$

where ω is the pion laboratory energy. The amplitude D is defined as

$$D(\nu, t) = A(\nu, t) + \nu B(\nu, t) \tag{14}$$

and its imaginary part can be related to the total cross section through the optical theorem, $\operatorname{Im} D(\omega, t = 0) = k_{\text{lab}}\, \sigma$. The isoscalar (+) and isovector (−) amplitudes $D^\pm$ can be written in terms of the amplitudes in the physical channels as

$$D^\pm = \frac{1}{2}(D_{\pi^- p} \pm D_{\pi^+ p}). \tag{15}$$

4 A low-energy theorem

Chiral symmetry allows us to write at the Cheng-Dashen point, i.e. at $(\nu = 0, t = 2M_\pi^2)$,

$$\Sigma \doteq F_\pi^2 \bar{D}^+(\nu = 0, t = 2M_\pi^2) = \sigma(2M_\pi^2) + \Delta_R, \tag{16}$$

where F_π is the pion decay constant, $\bar{D}^+$ is the isoscalar D-amplitude with the pseudovector Born term subtracted and Δ_R is the remainder, which is formally of the order M_π^4 [6]. To one-loop in ChPT ($\mathcal{O}(q^3)$) [7]

$$\Delta_R = 0.35 \text{ MeV}. \tag{17}$$

One-loop in HBChPT ($\mathcal{O}(q^4)$) gives the upper limit [8]

$$\Delta_R \simeq 2 \text{ MeV} \tag{18}$$

and here it is notable that no logarithmic contribution to order M_π^4 appears. This allows us to write

$$\Sigma \simeq \sigma(2M_\pi^2). \tag{19}$$

What remains to be fixed in order to determine the σ is the form factor difference

$$\Delta_\sigma = \sigma(2M_\pi^2) - \sigma(0). \tag{20}$$

ChPT to one loop gives [7]

$$\Delta_\sigma \simeq 5 \text{ MeV}. \tag{21}$$

Dispersion analysis yields [9]

$$\Delta_\sigma = 15.2 \pm 0.4 \text{ MeV}. \tag{22}$$

Becher and Leutwyler obtain [4]

$$\Delta_\sigma = 14.0 \text{ MeV} + 2M^4 \bar{e}_2, \tag{23}$$

where $\bar{e}_2$ is a renormalized coupling constant appearing in the $\mathcal{L}_{\text{N}}^{(4)}$ lagrangian. The constant $\bar{e}_2$ is expected to be small [4].

5 The Σ-term

Inside the Mandelstam triangle it is convenient to employ the subthreshold expansion [10], where $\bar{D}^+$ is expanded in powers of ν^2 and t

$$\bar{D}^+ = d_{00}^+ + d_{10}^+ \nu^2 + d_{01}^+ t + d_{20}^+ \nu^4 + d_{11}^+ \nu^2 t + \cdots \tag{24}$$

The curvature term Δ_D is defined as

$$\Sigma = F_\pi^2 (d_{00}^+ + 2M_\pi^2 d_{01}^+) + \Delta_D \equiv \Sigma_d + \Delta_D \tag{25}$$

and it is dominated by the $\pi\pi$ cut giving the result [9]

$$\Delta_D = 11.9 \pm 0.6 \text{ MeV}. \tag{26}$$

The linear part Σ_d is a sensitive quantity due to the cancellation of the d_{00}^+ and d_{01}^+ pieces in

$$\Sigma_d(\text{A}) = (-91.3 + 138.8) \text{ MeV} \simeq 48 \text{ MeV} \tag{27}$$

$$\Sigma_d(\text{B}) = (-94.5 + 144.2) \text{ MeV} \simeq 50 \text{ MeV} \tag{28}$$

corresponding to the two solutions (A and B) discussed in [5]. These numbers lead to $\Sigma \simeq 60$ MeV, which is consistent with the old result of Koch [11] $\Sigma = 64 \pm 8$ MeV based on hyperbolic dispersion relations.

6 The strangeness content of the nucleon

Putting all these pieces together leads to a determination of the strangeness content of the proton

$$\frac{\hat{\sigma}}{1-y} = \Sigma - \Delta_R - \Delta_\sigma \tag{29}$$

and numerically with the solution A

$$\frac{35 \text{ MeV}}{1-y} = (60 - 2 - 15) \text{ MeV}, \tag{30}$$

which gives $y \simeq 0.2$ with a sizeable error. This value of y corresponds to about 130 MeV in the proton mass being due to the strange sea.

7 Partial wave analysis

The analysis discussed in Sect. 5 was based on the KH80 solution of the Karlsruhe group [12]. The data basis used there contained mainly pre-meson-factory-era data and, therefore, it is of great interest to perform a new analysis with the new data in the spirit of the Karlsruhe group incorporating fixed-t constraints. This would hopefully help in fixing the value of Σ more accurately. In the forward direction it is feasible to solve the dispersion relations directly, but for $t < 0$ it is more practical to use the expansion method [10]. E.g., for the C^+ amplitude ($C = A + \nu/(1 - t/4m^2)B$)

$$C^+(\nu, t) = C_N^+(\nu, t) + H(Z, t) \sum_{n=0}^{N} c_n^+ Z^n,$$

where $C_N^+(\nu, t)$ is the Born term, the function H is adjusted to the asymptotic behaviour of the amplitude and Z is the conformal mapping

$$Z(\nu^2, t) = \frac{\alpha - \sqrt{\nu_{th}^2 - \nu^2}}{\alpha + \sqrt{\nu_{th}^2 - \nu^2}}, \tag{31}$$

where $\nu_{th} = \mathrm{M}_\pi + t/4m$ and α is a real parameter. The convergence and smoothing is taken care of by a convergence test function

$$\chi_T^2 = \lambda \sum_{n=0}^{N} c_n^2 (n+1)^3,$$

which is one component in the χ^2 expression to be minimized. Other contributions include χ^2_{DATA} and χ^2_{PW}, where the latter is calculated from the previous iteration of the partial wave solution.

To demonstrate the working of the expansion method at $t = 0$ with N=40, Figs. (1) and (2) display $\mathrm{Re}\, C^+(\omega, t = 0)$ and $\sigma^T_{\pi^+ p}$. For the real part of the C^+-amplitude there are three data points at low energy as input and they fix the subtraction constant appearing in the dispersion relation for the C^+.

The VPI/GWU group has recently published a partial wave analysis [13], which does employ fixed-t constraints. The publication does not, however, give a value for the Σ, but indications are [14] that the number could be 20-30 % larger than the numbers quoted above.

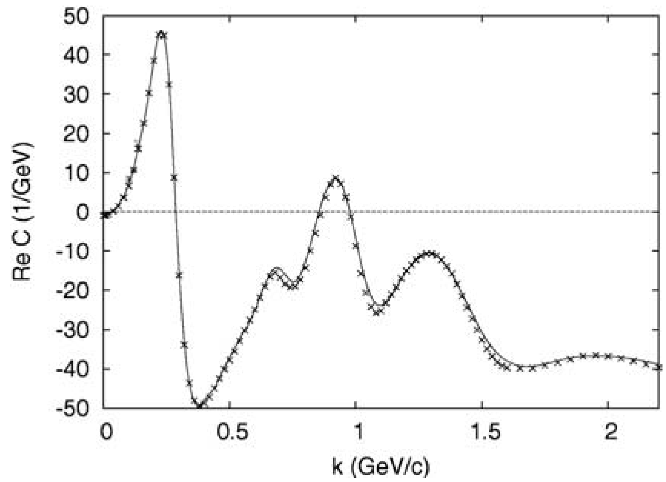

Fig. 1. The real part of the C^+-amplitude. The *crosses* refer to the tabulated values in [10]

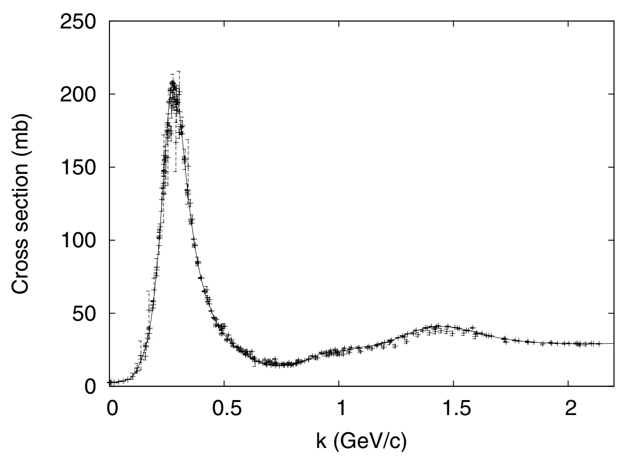

Fig. 2. The $\pi^+ p$ total cross section

8 The relation $\Sigma \leftrightarrow$ threshold

The issue of relating the Σ to the values of threshold parameters is an old one [15]. In general, Σ can be expressed in terms of the threshold parameters [16]

$$\Sigma = F_\pi^2 [L(a_l, \tau) + (1 + \frac{M_\pi}{m}) \tau J^+] + \delta, \tag{32}$$

where $L(a_l, \tau)$ is a linear combination of the threshold parameters a_l, τ is a free parameter to single out individual scattering lengths and J^+ is the integral over the isoscalar combination of the total cross section. The remainder, δ, contains contributions from the Born term, the Δ and the loop corrections. The approach of Altarelli et al. [15] corresponds to choosing $\tau = -1$, but without loops. However, at present, one has to rely on dispersion methods to extract the threshold parameters anyway, so the value of any such formula is limited.

9 Lattice results

ChPT permits a study of the quark mass dependence of the nucleon mass. This makes it possible to have a connection to the lattice data, where, currently, only unphysically high quark masses can be dealt with. New accurate data from the CP-PACS, JLQCD and QCDSF-UKQCD collaborations (dynamical quarks, two flavours) give [17]

$$\sigma = 49 \pm 3\ \mathrm{MeV} \tag{33}$$

to $\mathcal{O}(q^4)$ in ChPT. Another approach including the leading nonanalytic and next-to-leading nonanalytic behaviour yields [18]

$$\sigma = 35 - 73\ \mathrm{MeV}. \tag{34}$$

10 Conclusions

The challenge at present seems to be in determining Σ. That involves a number of questions

– one has to deal with conflicting sets of data,

– one has to rely on the Tromborg [19] formalism for the electromagnetic corrections even though there are indications [20] that further improvements in this sector should be incorporated,

– the extrapolation from the low-energy region to the Cheng-Dashen point could, to some extent, be sensitive to the d-waves, which otherwise cannot be fixed with the low-energy scattering information [21].

The new direction with the lattice calculations is gradually getting very interesting as far as the sigma-term is concerned. However, further improvements, i.e. smaller m_q-values, will still be needed.

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